

Perturbative QCD estimation of the $B \rightarrow K^* + \gamma$ branching ratio

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Abstract

Working in a perturbative QCD model approach, we obtain the essential form factor of the radiative transition $B \rightarrow V^* \gamma$ and estimate the branching ratio $BR(B \rightarrow K^* \gamma)$. The results are determined by a parameter expressing the momentum distribution in the B - meson wave function. Our estimations are compared to other theoretical predictions as well as to experimental data.

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1 Introduction

Studies of rare B decays are very active now especially after CLEO II experiment reported the first direct evidence for one-loop flavor-changing neutral current diagram in $B \rightarrow K^*(892)\gamma$ [1]. The rare decays amplitudes proportional to V_{ub} as well as the so-called “penguin” processes are important for CP-violation measurements. As $b \rightarrow u$ is suppressed by the corresponding small value of CKM matrix element, the electromagnetic $b \rightarrow s$ transition brings the dominant contribution. Attempts in extending perturbative QCD effects in factorized diagrams dominated by a single gluon exchange with the spectator quark developed by Szczepaniak, Henley and Brodsky [2], have been made in calculating both semileptonic and nonleptonic B - decays with 0^- and 1^- mesons in final state [3, 4]. A comprehensive study of radiative rare decays, using the spin symmetry for heavy quarks combined with meson wave function models belong to Ali and Greub [5] as well as to Ali, Ohl and Mannel [6]. Considerations on radiative transitions of B mesons, with an up to date review of experimental results can be found in [7], the theoretical implications of $B \rightarrow K^*\gamma$ for the physics beyond the standard model being also pointed out.

Our work is organized as follows. After a brief review of the Szczepaniak, Henley and Brodsky model, which is basic for the rest of the material, in section 3 we put the amplitude of the radiative transition $B \rightarrow V^*\gamma$ in terms of form factors. In section 4 we start from the effective hamiltonian

$$H = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* C_7(m_b) \tilde{O}_7(m_B), \quad (1)$$

where $C_7(m_b)$ containing the effects of QCD corrections is the Wilson coefficient at $\mu = m_b$ and $\tilde{O}_7(m_B)$ is computed in Brodsky and Lepage perturbative QCD approach, replacing the usual one [6]. Then, the branching ratio of the two body decay $B \rightarrow K^*\gamma$, expressed as a function of the a - parameter characterizing the momentum distribution between the pair of quarks in the heavy B meson is calculated. Finally, we perform a comparison with other theoretical results and with the experimental limits.

2 Brief Review of the Model

Our calculations being done within the framework of perturbative QCD effects studied by Szczepaniak, Henley and Brodsky [2], it is worthwhile to summarize here their approach in estimating hadronic matrix elements of the transition currents. Assuming the factorization for the case of an exclusive nonleptonic decay of a heavy meson into two much lighter 0^- mesons and neglecting the final state interactions, they have written the corresponding decay amplitude as a convolution of a collinear irreducible hard-scattering quark-gluon amplitude T^μ and the mesons wave functions ϕ which, besides the spin factors [8], contain the fractional longitudinal momentum distribution of the collinear quarks. Distinct from the usual HQET [9, 10] the dominant contribution in $\alpha_s(Q^2 \sim M_B^2)$ is controlled by a single gluon exchange with the spectator. After factorization and neglecting the disconnected diagram whose contribution vanishes, the matrix element of the transition current between the two remaining hadronic states is put in terms of form factors. For a massive initial state, the decay amplitude is of order $\alpha_s(Q^2) \sim 0.38 (\Lambda_{QCD}^2 = 0.01 \text{ GeV}^2)$, even without including loop corrections.

Although this method has been developed for exclusive nonleptonic decays, we have applied it for both nonleptonic and semileptonic heavy meson decays, with 1^- and 0^- light mesons in the final state [3, 4] and the obtained results, compared with other theoretical predictions and experimental upper limits, have encouraged us to test its validity in estimating other decays, such as the radiative transitions. The first observation of the electromagnetic decay $B \rightarrow K^* \gamma$, reported in 1993 by CLEO II [1], has made this process of a real theoretical interest, due to its implications in the physics beyond the standard model [7]. Even the process is described by the one-loop flavor-changing neutral current diagram, it has been used factorization to estimate its branching ratio, although it is not decided whether factorization is a correct framework for the so-called penguin diagrams. However, in this case one has to take into account the short distance QCD corrections.

3 Radiative Transition Form Factors

The method described allows the calculation of the heavy-to-light radiative transition $B \rightarrow V^* \gamma$ in the same way as in the nonleptonic decays, by con-

sidering a single gluon exchange with the spectator (see fig.1).

Figure 1

To first order in α_s , the matrix element of the transition current can be computed as

$$T_\mu = g_s^2 \left\{ \text{Tr} \left[\bar{\phi}_V \sigma_{\mu\nu} (1 - \gamma_5) q^\nu \frac{\gamma \cdot k_B + m_B}{k_B^2 - m_B^2} \gamma_\alpha \phi_B \gamma^\alpha \frac{\lambda_a \lambda^a}{Q^2} \right] + \right. \\ \left. + \text{Tr} \left[\bar{\phi}_V \gamma_\alpha \frac{\gamma \cdot k_V}{k_V^2} \sigma_{\mu\nu} (1 - \gamma_5) q^\nu \phi_B \gamma^\alpha \frac{\lambda_a \lambda^a}{Q^2} \right] \right\}, \quad (2)$$

$\gamma \cdot k$ being $\gamma^\beta k_\beta$.

Here,

$$\phi_B(x) = \varphi_B(x) (\gamma \cdot P_B + m_B) \gamma_5 \quad (3)$$

is the wave function of the heavy B - meson with the mass m_B and the distribution amplitude [2, 3, 4]

$$\varphi_B(x) = \frac{f_B}{12} \frac{x^2(1-x)^2}{[a^2x + (1-x)^2]^2} \cdot \left\{ \int_0^1 \frac{x^2(1-x)^2}{[a^2x + (1-x)^2]^2} dx \right\}^{-1} \quad (4)$$

where $a \sim 0.05 - 0.1$ is related to the maximum of $\varphi_B(x)$ in the B - meson, and

$$\phi_V = \varphi_V(y) (\gamma \cdot P_V) (\gamma \cdot \varepsilon) \quad (5)$$

with

$$\varphi_V(y) = \frac{f_V}{12} y(1-y) \left\{ \int_0^1 y(1-y) dy \right\}^{-1} \quad (6)$$

denotes the wave function of the light vector meson of polarization vector ε^μ . The Tr means trace over spin, flavor and color indices and integration over momentum fractions.

We put (2) in the form

$$T_\mu = i\varepsilon_{\mu\nu\alpha\beta} \varepsilon^\nu q^\alpha P_V^\beta V + \frac{\varepsilon_\mu}{2} m_B^2 A_1 - (\varepsilon \cdot q) (P_V)_\mu A_2 - \frac{(\varepsilon \cdot q)}{2} q_\mu A_3 \quad (7)$$

pointing out the a - dependent expression of the essential form factor as being:

$$|V| = |A_1| = |A_2| = 64 \frac{g_s^2}{m_B^2} \left\{ \int_0^1 \frac{\varphi_B(x)}{1-x} dx \int_0^{1-a} \frac{\varphi_V(y)(2-y)}{(1-y)^2} dy \right\}, \quad (8)$$

where we have kept only the first order in $1 - x \sim a$.

For $B \rightarrow K^* \gamma$ the numerical values of the form factor are from 0.25 (for $a = 0.1$) to 0.6 (for $a = 0.05$).

4 Branching Ratio $BR(B \rightarrow K^* + \gamma)$

In order to make an estimation for the exclusive $B \rightarrow K^* \gamma$ branching ratio

$$BR(B \rightarrow K^* \gamma) = \frac{\Gamma(B \rightarrow K^* \gamma)}{\Gamma_{tot}}, \quad (9)$$

we start from the effective hamiltonian (1) with the operator $\tilde{\mathcal{O}}_7$ of the form

$$\tilde{\mathcal{O}}_7 = \frac{e}{16\pi^2} m_B \eta^\mu T_\mu, \quad (10)$$

where η^μ is the polarization vector of the photon and T_μ is (2), and we get for $\Gamma(B \rightarrow K^* \gamma)$ the expression:

$$\Gamma(B \rightarrow K^* \gamma) = \frac{\alpha G_F^2 m_B^5}{128\pi^4} |V_{tb} V_{ts}^*|^2 |C_7(m_b)|^2 |V|^2 \quad (11)$$

In order to numerically estimate the $BR(B \rightarrow K^* \gamma)$ we take as input the following numerical values [5]:

$$\begin{aligned} r_u &\approx 7, \quad r_c \approx 3, \quad |V_{ub}| \approx 0.0075, \quad |V_{cb}| = |V_{ts}| \approx 0.045, \\ G_F &= 1.16637 \times 10^{-5} \text{GeV}^{-2} \\ W_{eff}^2 &\simeq M^2 - 2M \cdot p_F \cdot 1.13, \text{ with the parameter } p_F \approx 0.3 \text{ GeV} \end{aligned} \quad (12)$$

in the b quark width

$$\Gamma_{tot} = \frac{W_{eff}^5 G_F^2}{192\pi^3} (r_u |V_{ub}|^2 + r_c |V_{cb}|^2) \quad (13)$$

Thus, we come to an a - dependent branching ratio whose values are from around $\simeq 0.59 \times 10^{-5}$ (for $a = 0.1$) to $\simeq 3.43 \times 10^{-5}$ (for $a = 0.05$).

A comparison to other theoretical models predictions and to experimental data restricts the range of this parameter. Thus, the estimation $BR(B \rightarrow K^*(892)\gamma) = (1.4 - 4.9) \times 10^{-5}$ [6] corresponds to the a - parameter in the

range $a = 0.072 - 0.043$, while a comparison to the branching ratio reported by CLEO II [7] sets the upper limit for a at about 0.06.

Our calculations have been made in the assumption that the mass of K^* can be neglected in comparison to the mass of the B meson. As mentioned in [2], this has brought considerable simplifications. Of course, for comparing the branching ratios for different K^* mesons one has to take into account besides \tilde{O}_7 , a second operator \tilde{O}'_7 proportional to m_K and to replace the wave function (5-6) with a type (3-4) one, but with different a . Thus, the branching ratios will depend on three parameters, namely a, a' and the mass parameter $z = m_K/m_B$. However, testing whether Brodsky and Lepage approach is applicable in these assumptions is beyond the aim of the present paper.

5 Conclusions

We have developed an extension of our earlier applications [3, 4] of the Szczepaniak, Henley and Brodsky procedure of perturbative QCD calculations of hadronic current matrix element [2] to the case of the rare B decay $B \rightarrow K^*\gamma$. The branching ratio of this process is shown as a function of the parameter a , having numerical values from around $\simeq 0.59 \times 10^{-5}$ to $\simeq 3.43 \times 10^{-5}$ for $a \in [0.05, 0.1]$, the other theoretical model estimation [6] corresponding to $a \leq 0.072$, while the experimental lower limit of about 2.1×10^{-5} imposing $a \leq 0.06$.

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Figure 1: The contributing diagrams in the hard scattering amplitude T_μ